Find $\frac{d}{dx} x^5 \cosh^{-1}(x^3)$. Simplify your final answer as a single fraction.

SCORE: ______ / 4 PTS

You may use the derivatives of any hyperbolic or inverse hyperbolic functions from your textbook without proving them.

$$5x^4 \cosh^{-1} x^3 + x^5 \frac{3x^2}{\sqrt{(x^3)^2 - 1}}$$

$$5x^4 \cosh^{-1} x^3 + \frac{3x^7}{\sqrt{x^6 - 1}}$$

1 1 2

If $\coth x = -7$, find $\sinh x$. ALTERNATE SOLUTION ON NEXT PAGE SCORE: ______ / 4 PTS

$\cosh^2 x - \sinh^2 x = 1$

$\coth^2 x - 1 = \operatorname{csch}^2 x$ ①

$48 = \operatorname{csch}^2 x$ ①

$\operatorname{csch} x = \pm 4\sqrt{3}$ $\longrightarrow$

$= -4\sqrt{3}$ ½

$\coth x = \dfrac{\cosh x}{\sinh x}$

SINCE $\coth x < 0$

AND $\cosh x > 0$ (FOR ALL x),

SO $\sinh x < 0$

AND $\operatorname{csch} x < 0$ ½

$\sinh x = -\dfrac{1}{4\sqrt{3}} = -\dfrac{\sqrt{3}}{12}$ ½ ½

If $\coth x = -7$, find $\sinh x$.

SCORE: ______ / 4 PTS

$\tanh x = -\frac{1}{7}$

$\frac{1}{2}$ POINT EACH

$\cosh^2 x - \sinh^2 x = 1$

$\frac{\sinh x}{\cosh x} = -\frac{1}{7}$

$1 - \tanh^2 x = \operatorname{sech}^2 x$

$\frac{48}{49} = \operatorname{sech}^2 x$

$\sinh x = -\frac{1}{7}\cosh x$

$\frac{4\sqrt{3}}{7} = \operatorname{sech} x > 0$

$= -\frac{1}{4\sqrt{3}} = -\frac{\sqrt{3}}{12}$

$\frac{7}{4\sqrt{3}} = \cosh x$

Find $\lim\limits_{x\to\infty} \coth x$ **algebraically**. **SCORE: ______ / 3 PTS**

$$\underbrace{\lim_{x\to\infty} \frac{e^{2x}+1}{e^{2x}-1}}_{1 \quad \frac{\infty}{\infty}} = \underbrace{\lim_{x\to\infty} \frac{\cancel{2}e^{\cancel{2}x}}{\cancel{2}e^{2x}}}_{\text{L'HÔPITAL'S RULE}} \; 1 = \underbrace{1}_{} \; 1$$

For this question, you may use the formulae for $\frac{d}{dx}\sinh x$, $\frac{d}{dx}\cosh x$ and/or $\frac{d}{dx}\tanh x$ without proving them. **SCORE: _____ / 8 PTS**

If you need to use the formula for the derivative of any other hyperbolic function, you must prove it.

[a] **Without using the logarithmic formula for $\tanh^{-1} x$**, prove the formula for $\frac{d}{dx}\tanh^{-1} x$.

$y = \tanh^{-1} x$

(1) $x = \tanh y$,

$(1\frac{1}{2})$ $1 = \operatorname{sech}^2 y \frac{dy}{dx}$,

$1 = (1 - \tanh^2 y)\frac{dy}{dx}$

$1 = (1 - x^2)\frac{dy}{dx}$

$(1\frac{1}{2})$ $\frac{dy}{dx} = \frac{1}{1-x^2}$,

$\cosh^2 y - \sinh^2 y = 1$

(1) $1 - \tanh^2 y = \operatorname{sech}^2 y$,

[b] **Without using the exponential formula for sech x**, prove the formula for $\frac{d}{dx}\operatorname{sech} x$.

$\frac{d}{dx}\frac{1}{\cosh x} = -(\cosh x)^{-2}\sinh x$, (2)

$= -\frac{1}{\cosh x}\frac{\sinh x}{\cosh x}$

$= -\operatorname{sech} x \tanh x$, (1)

Prove the logarithmic formula for $\sinh^{-1} x$ given in your textbook. **SCORE: ______ / 5 PTS**

NOTE: This is NOT a question about derivatives.

$y = \sinh^{-1} x$

1 $x = \sinh y = \dfrac{e^y - e^{-y}}{2}$ LET $z = e^y$, SO $z > 0$ $\frac{1}{2}$

$\frac{1}{2}$ $x = \dfrac{z - \frac{1}{z}}{2}$

$2x = z - \frac{1}{z}$

$2xz = z^2 - 1$

$\frac{1}{2}$ $0 = z^2 - 2xz - 1$

1 $z = \dfrac{2x \pm \sqrt{4x^2+4}}{2} = x \pm \sqrt{x^2+1}$ $\frac{1}{2}$

$e^y = z = x + \sqrt{x^2+1}$ $\frac{1}{2}$

$\sinh^{-1} x = y = \ln\left(x + \sqrt{x^2+1}\right)$

$\frac{1}{2}$